Appendix A: Development of epidemic mathematical transmission model

An epidemic mathematical transmission model of HIV and HCV among Australian IDUs was formulated to dynamically describe the change in the number of people in the population over time according to disease states. The model considered heterogeneity in injecting behaviour reported by the Illicit Drug Reporting System (IDRS) [56-62]: IDUs who did not inject in the last month, injected weekly or less, injected more than weekly, injected once daily, injected two to three times per day, and injected more than three times per day. The frequency of sharing injecting equipment, number of people with whom equipment is shared, number of times each syringe is used before it is disposed, and frequency with which syringes and other injecting equipment (e.g. spoons, tourniquets, etc) are cleaned before reuse, and the efficacy of cleaning equipment contaminated with HIV or HCV were all factored into the model's calculation of the per capita rate of IDUs becoming infected. The model also tracked the entry of new injectors into the population and the rate of ceasing injecting behaviour, while also matching the assumed dynamic number of IDUs in the population over time. Drugrelated, disease-related, and background death rates were also included. All parameter values were estimated based on exhaustive searching of the relevant literature and available data from Australian reports and databases (see Table B.1).

Data were also stratified by each Australian state and territory as well as Aboriginal and Torres Strait Islander populations. The numbers of IDUs in each jurisdiction were included, based on various indicators, along with the dynamic number of sterile syringes distributed by NSPs to these populations over time. We considered different syringe coverage rates within the IDU populations.

Force of infection and analysis of 'static' incidence

Based on these factors, we formulated a mathematical expression for the 'force of infection', which refers to the dynamic rate at which susceptible individuals become infected. The force of infection used in this analysis was developed by first considering a static and homogeneous population of N IDUs and was then adapted to include heterogeneous and dynamic features. In a homogenous population, if each IDU injects an average of n times per year, a proportion, s, of IDUs share their syringes with others in a proportion, q, of their injections, and sharing occurs in groups of m people, then the total number of 'sharing

events' in the population per year is $\frac{Nnsq}{m}$. The total number of expected transmissions will be this number multiplied by the average number of transmissions per 'sharing event'.

If the prevalence in the population is *P*, then the probability of *r* infected people in a sharing group of size $m \operatorname{is} \binom{m}{r} P^r (1-P)^{m-r}$, using standard binomial theory. If the group members inject using the shared syringe in random order, then an average of $\frac{m-r}{r+1}$ uninfected people will inject before the first infected person (and between each infected person). Therefore, in each sharing event an average of $m - \frac{m-r}{r+1} - r = \frac{rm-r^2}{r+1}$ uninfected people will use a syringe after an infected person has used it. If a shared syringe is used δ_s times before disposal then m/δ_s syringes are used in each 'sharing event' and the average number of uninfected people in the group to use the same syringe after an infected person becomes $\frac{rm-r^2}{r+1} \frac{\delta_s}{m}$. If the probability of infection from a contaminated syringe per use is β , but transmission is reduced by an effectiveness of ε_c through syringe cleaning and cleaning occurs before a proportion, p_c , of shared injections, then each susceptible person could acquire infection with probability $(1-p_c\varepsilon_c)\beta$ if using a contaminated syringe. Therefore, the expected number of transmission is a given sharing group (or probability of a transmission occurring) is

$$\frac{\delta_{S}\beta(1-p_{c}\varepsilon_{c})}{m}\sum_{r=1}^{m-1}\binom{m}{r}P^{r}(1-P)^{m-r}\frac{rm-r^{2}}{r+1}$$

Then the total number of transmissions expected each year, or incidence (I), is

$$I = \frac{Nnsq\delta_{S}\beta(1-p_{c}\varepsilon_{c})}{m^{2}} \sum_{r=1}^{m-1} {m \choose r} P^{r} (1-P)^{m-r} \frac{rm-r^{2}}{r+1}.$$
 (1)

The reader is referred to [63] for details of thorough analyses of this static expression, applied to Australian IDUs. Below are summary results from these analyses.

The expected reproduction ratio, R, per IDU was calculated for HIV and HCV as a function of the average duration of injecting post-seroconversion (Figure A.1): if each IDU injects for an average of D years after seroconversion, then the average number of secondary cases per IDU is

$$R = \frac{Dnq\delta_{s}\beta(1-p_{c}\varepsilon_{c})}{m^{2}P} \sum_{r=1}^{m-1} {m \choose r} P^{r} (1-P)^{m-r} \frac{rm-r^{2}}{r+1}.$$
(2)

An epidemic is sustained if *R* is greater than one [64], implying that each infected person is associated with at least one secondary transmission on average. It was found that the threshold duration of injecting post-seroconversion required to sustain an epidemic is 11.6 (7.0-22.4, IQR) years for HIV and 2.3 (1.8-3.2, IQR) years for HCV (Figure A.1). Based on behavioural data [54, 65] it is reasonable to assume that the average duration of injecting post-HCV seroconversion is ~ten years. This is considerably greater than the threshold of 2.3 years required to control HCV incidence. In contrast, the duration of injecting for HIV-infected IDUs, post-seroconversion, is assumed to be much less than for HCV (less than ten years) and thus less than the critical 11.6 years required to control HIV incidence.

Figure A.1: The average number of secondary cases of HIV (orange) and HCV (blue) transmission per IDU versus the duration of injecting post-seroconversion. The solid lines refer to median simulations and the dashed line refers to one secondary infection.



To identify factors that could provide effective targets for intervention a sensitivity analysis was conducted, by means of calculating partial rank correlation coefficients [40] between incidence and the sampled model parameters (results not shown). It was determined that the number of times each syringe is used before disposal is the most sensitive behavioural factor in determining the incidence of both HIV and HCV infection, followed by the percentage of injections that are shared. Therefore, the expected change in incidence for HIV and HCV was

investigated in relation to the frequency of shared injections and the average number of times each syringe is used (Figure A.2).

Figure A.2: The simulated number of annual (a) HIV and (b) HCV transmissions among IDUs in Australia versus the percentage of injections that are shared and the average number of times each syringe is used before disposal. The dashed lines refer to current levels of sharing and syringe use.



The number of times each syringe is used may be decreased by greater dissemination of sterile syringes through NSPs. The number of syringes distributed through NSPs has remained relatively constant over the last decade (see Table B.1), suggesting that saturation levels have been reached. However, there is also reason to believe that there are opportunities for public sector NSP services to increase client reach. It is difficult to estimate the proportion of all IDUs that access NSPs, however, the recent National Drug Strategy Household Survey revealed that only 51% of those who had injected in the last 12 months usually obtained their injecting equipment from public sector NSPs [66]. Structural and policy factors may limit access to current NSP services. With the exception of pharmacy-based services, few NSPs operate into the evening or are open on weekends. Whilst syringe dispensing machines operate 24 hours a day, these not are operational throughout Australia. There are also limits on the quantity and range of syringes freely available at some NSP services. Secondary exchange of sterile needles and syringes (from one IDU to another) is prohibited in most states and territories, and there are some locations where there is demand for NSP, but where services are not well developed. These factors suggest that syringe distribution in Australia is limited by supply rather than demand, and that increased coverage is possible.

If *K* syringes are distributed each year and a proportion ω of all syringes are not used, then the number of syringes distributed that are used is $P(1-\omega)$. The number of syringes used for individual injecting episodes among non-sharing IDUs is $\frac{nN(1-s)}{\delta_p}$. Similarly, the total number of syringes used for individual injecting among all sharing IDUs is $\frac{n(1-q)sN}{\delta_p}$ and

the total number of syringes used in sharing events is $\frac{nqsN}{\delta_s}$. Therefore,

$$K(1-\omega) = \frac{nN(1-s)}{\delta_p} + \frac{n(1-q)sN}{\delta_p} + \frac{nqsN}{\delta_s} = \frac{nN}{\delta_p\delta_s} \Big[\delta_s - sq(\delta_s - \delta_p) \Big]$$
(3)

defines a relationship between the total number of syringes distributed and the use of syringes in this mathematical model (equation 2). Changes in the number of syringes distributed are likely to change any, or all, of the following factors in a way that is consistent with equation 3: the proportion of syringes that remain unused (ω) , the proportion of injections that are shared (q), or the average number of times each syringe is used (in shared (δ_s) or individual (non-shared) injections (δ_p)). Changes to ω and δ_p will not influence transmission levels but changes to q and δ_s could potentially result in large reductions in incidence. It could be speculated that increased syringe coverage is most likely to influence a decrease in the number of injections per syringe (for both personal and shared syringes). Therefore, equation 3 was used to estimate the change in the average number of injections per syringe used in both individual and shared injections, assuming the same percentage increase or decrease for both, according to a change in the total number of syringes distributed. The new values for the usage per syringe (δ_P and δ_S) were then used in equation (1), and all other parameters were sampled independently from their original distributions as defined in Table B.1. This was used to estimate the expected incidence of HIV and HCV based on changes in syringe distribution (Figure A.3). It should be noted that very large increases in syringe distribution are likely to be infeasible and unrealistic. It is also important to acknowledge that other relationships between incidence and syringe distribution could be expected if syringe distribution affected other factors in equation 3. However, Figure A.3 does demonstrate that it greater NSP distribution of syringes may lead to reductions in incident cases of HIV and HCV and that if there was a decline in syringe distribution through NSPs then significant increases in incidence could be expected. It is likely that the provision of NSP services has contained the HIV epidemic among IDUs.

Figure A.3: Scatter plots of the simulated number of annual (a) HIV and (b) HCV transmissions among IDUs in Australia versus the number of sterile syringes distributed in Australia are shown, assuming that syringe distribution changes the average number of times each syringe is used before disposal. The blue dots are results from 1000 simulations, the red curves represent the median parameter values, and the black dashed lines refer to current levels of syringe distribution.



Dynamic transmission model

The model used in the analyses of this report extends the 'static' mathematical expression (equations 1 to 3) by including time-dependent parameter estimates for all demographic parameters and simulating the dynamic model-based prevalence of HIV and HCV in the population. Various assumptions about the role NSPs and syringe distribution among heterogeneous groups of IDUs were also considered.

Furthermore, an extensive natural history model of HIV and HCV monoinfection or coinfection was developed to dynamically track the number of people in each HIV and HCV health state. A schematic diagram of compartments of the HIV and HCV transmission model for IDUs in Australia is presented in Figure A.4. The change in the number of people in each compartment was tracked mathematically by formulating a system of 473 ordinary differential equations, one for each compartment. One compartment represents IDUs who are not infected with HIV or HCV. Fifteen compartments represent IDUs who are monoinfected with HCV: in acute stage, fibrosis stages F0, F1, F2, F3, F4, and for each of these, whether they are untreated or receiving treatment. People infected with HCV who have advanced

fibrosis can progress to clinical outcomes of liver failure, hepatocellular carcinoma, or may receive a liver transplant. It is assumed that individuals that progress to these three clinical outcomes no longer inject drugs.

Sixteen compartments represent IDUs who are monoinfected with HIV: individuals who become HIV-infected are initially untreated and are assumed to have a $CD4^+$ T cell count above 500 cells per µl, then will progress in their disease through categories according to $CD4^+$ T cell levels (350-500 cells per µl, 200-350 cells per µl, and <200 cells per µl); HIV-infected individuals may initiate antiretroviral therapy (for each $CD4^+$ T cell category the number of individuals on effective first-line treatment, treatment failure, or effective second-line treatment are also tracked). This model also tracks potential co-infection of HIV and HCV, including all possible combinations of HIV and HCV disease states; however, it is assumed that HIV-infected individuals with CD4 counts less than 350 cells per µl and on antiretroviral therapy will not also receive treatment for their HCV infection at the same time.

Thus, 205 ordinary differential equations are used to describe the co-infection of HIV and HCV among IDUs. This model also tracks the disease progression of individuals who have stopped injecting drugs but are infected with HIV and/or HCV. The number of equations is then doubled, plus the equation for uninfected but susceptible IDUs, leading to a total of 473 ordinary differential equations, one for each model compartment, to describe the number of people in each health state. The flows in the number of people between these compartments are due to biological, behavioural, clinical, or epidemiological parameters (specified in detail in Appendix B).



Figure A.4: Schematic diagram of compartments of the HIV and HCV transmission model for IDUs in Australia

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The mathematical description of the model is below:

Uninfected IDUs

Change in Force of HCV infection uninfecteds Entry into Force of Background Drug-related Exit $+ \overset{Ex.i}{\xi}$ death population HIV infection dSdeath ~~ ~ $\overline{\lambda}_{_{HIV}}$ $\lambda_{_{HCV}}$ π S μ μ_D ++=dt Viral clearance Viral clearance Viral clearance Viral clearance from treatment (F0) from treatment (F2) from treatment (F1) from treatment (acute) $\widetilde{\gamma^M_A v^M_A I^{MT}_A}$ 1 1 1 $\gamma^M_{F1} v^M_F I^{MT}_{F1}$ $\widetilde{\gamma_{F2}^M} \widetilde{\nu_F^M} I_{F2}^{MT}$ $\widetilde{\gamma^M_{F0}} v^M_F I^{MT}_{F0}$ ++++Viral clearance from treatment (F3) Viral clearance from treatment (F4) Spontaneous viral clearance (acute) + $\gamma_{F4}^M v_F^M I_{F4}^{MT}$ $+ \gamma_{F3}^{M} \nu_{F}^{M} I_{F3}^{MT}$ $\psi_{M}I_{A}^{M}$ +

HCV-infected individuals

Change in

Change in
acute infecteds

$$\frac{dI_{A}^{M}}{dt} = \overset{New infections}{\lambda_{HCV}S} + \overbrace{(1-\gamma_{A}^{M})}^{Cease treatment} \bigvee_{(acute)}^{A} I_{A}^{MT} - \left(\begin{array}{c} Background & Drug-related & Exit rate \\ death & death & death \\ \mu & + & \mu_{D} & + \overleftarrow{\xi} \end{array} \right)$$

$$+ \overbrace{\lambda_{HIV}}^{Force of} + \overbrace{\tau_{A}^{M}}^{Progress to} & Commence \\ + \overbrace{\lambda_{HIV}}^{Commence} + \overbrace{\tau_{A}^{M}}^{M} + \overbrace{\eta_{A}}^{M} \right) I_{A}^{M}$$

$$\frac{dI_{F0}^{M}}{dt} = \frac{\Gamma_{A}^{M}I_{A}^{M}}{\tau_{A}^{M}I_{A}^{M}} + (1 - \gamma_{F0}^{M})\nu_{F}^{M}I_{F0}^{MT} - \begin{pmatrix}Background & Drug-related & Exit \\ death & death & rate \\ \mu & \mu_{D} & + \tilde{\zeta} \\ \mu & \mu_{D} & + \tilde{\zeta} \\ \mu & \mu_{D} & + \tilde{\zeta} \\ \mu & \mu_{D} & \mu_{D} & \mu_{D} \\ \mu & \mu_{D} & \mu_{D} \\ \mu & \mu_{D} & \mu_{D} & \mu_{D} & \mu_{D} \\ \mu & \mu_{D} & \mu_{D} & \mu_{D} \\ \mu & \mu_{D} & \mu_{D} & \mu_{D} & \mu_{D} \\ \mu & \mu_{D} & \mu_{D} & \mu_{D} & \mu_{D} \\ \mu & \mu_{D} & \mu_{D} & \mu_{D} & \mu_{D} & \mu_{D} \\ \mu & \mu_{D} & \mu_{D} & \mu_{D} & \mu_{D} & \mu_{D} \\ \mu & \mu$$

$$\frac{dI_{F1}^{M}}{dt} = \tau_{F01}^{M} I_{F0}^{M} + (1 - \gamma_{F1}^{M}) v_{F}^{M} I_{F1}^{MT} - \begin{pmatrix} Background & Drug-related & Exit \\ death & death & rate \\ death & death & rate \\ \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \xi \\ \hline \mu & + \mu_{D} & + \xi \\ \hline \mu & + \xi \\$$

$$\frac{\widetilde{dI}_{F2}^{M}}{\widetilde{dt}} = \tau_{F1}^{M} + (1 - \gamma_{F2}^{M}) \nu_{F}^{M} I_{F2}^{MT} - \left(\frac{Background}{death} & Drug-related \\ \frac{death}{death} & \frac{death}{death} & rate \\ \widetilde{\mu} & + \tilde{\mu}_{D} & + \widetilde{\xi} \\ \end{array}$$

Force of Commence Progress to
HIV infection treatment (F2) F3
+
$$\lambda_{HIV}$$
 + η_{F2} + τ_{F2}^{M}

$$\frac{\widetilde{dI}_{F3}^{M}}{dt} = \widetilde{\tau}_{F2}^{M} I_{F2}^{M} + (1 - \gamma_{F3}^{M}) v_{F}^{M} I_{F3}^{MT} - \begin{pmatrix} Background & Drug-related & Exit \\ death & death & rate \\ \mu & + & \mu_{D} & + \overleftarrow{\xi} \end{pmatrix}$$

Force of Commence Progress to
HIV infection treatment (F3) F4
+
$$\lambda_{HIV}$$
 + η_{F3} + τ_{F3}^{M} I_{F3}^{M}

$$\frac{\widetilde{dI}_{F4}^{M}}{dt} = \widetilde{\tau}_{F3}^{M} I_{F3}^{M} + (1 - \gamma_{F4}^{M}) v_{F}^{M} I_{F4}^{MT} - \begin{pmatrix} Background & Drug-related & Exit \\ death & death & rate \\ \widetilde{\mu} & + & \widetilde{\mu}_{D} & + & \widetilde{\xi} \\ \end{bmatrix}$$

Force of Commence Progress to Progress to
HIV infection treatment (F4) liver failure
$$\mathcal{HCC}$$

+ λ_{HIV} + η_{F4} + τ_{F4LF}^{M} + τ_{F4HCC}^{M}

Change in acute

$$\frac{dI_{A}^{MT}}{dt} = \eta_{A}I_{A}^{M} - \left(\begin{matrix} Background Drug-related Exit Force of death death death rate HIV infection \\ \mu & + \mu_{D} & + \xi + \lambda_{HIV} \end{matrix}\right)$$

$$+\underbrace{\overbrace{(I-\gamma_{A}^{M})}^{Cease treatment}}_{(F4)} + \underbrace{\overbrace{(I-\gamma_{A}^{M})}^{Viral clearance}}_{A} + \underbrace{\overbrace{\gamma_{A}^{M}}^{Progress to F0}}_{A} + \underbrace{\tau_{A}^{MT}}_{A} + \underbrace{\tau_{A}^{MT}}_{A} + \underbrace{\tau_{A}^{MT}}_{A}$$

Change in F0 infecteds on

$$\frac{dI_{F0}^{MT}}{dt} = \tau_{A}^{MT} I_{A}^{MT} + \eta_{F0} I_{F0}^{M} - \left(\begin{matrix} Background Drug-related Exit death rate \\ eath death rate \\ \mu + \mu_{D} + \xi \end{matrix}\right)$$

$$+ \overbrace{\lambda_{HIV}}^{Force of} + \underbrace{(1 - \gamma_{F0}^{M})\nu_{F}^{M}}_{W_{F}} + \underbrace{(1 - \gamma_{F0}^{M})\nu_{F}^{M}}_{W_{F}} + \underbrace{\gamma_{F0}^{M}\nu_{F}^{M}}_{W_{F}} + \underbrace{\tau_{F0}^{MT}}_{W_{F}} + \underbrace{\tau_{F0$$

Change in F1 infecteds on

$$\frac{dI_{F1}^{MT}}{dt} = \tau_{F0}^{MT} I_{F0}^{MT} + \eta_{F1} I_{F1}^{M} - \begin{pmatrix} Background & Drug-related & Exit \\ death & death & rate \\ \mu & + \mu_D & + \xi \end{pmatrix}$$

Force of
HIV infection
+
$$\lambda_{HIV}$$
 + $(1 - \gamma_{F1}^{M})v_{F}^{M}$ + $\gamma_{F1}^{M}v_{F}^{M}$ + τ_{F1}^{MT}

$$\frac{dI_{F2}^{MT}}{dt} = \tau_{F1}^{MT} I_{F1}^{MT} + \eta_{F2} I_{F2}^{M} - \left(\begin{matrix} Background & Drug-related & Exit \\ death & death & rate \\ \hline \mu & + & \mu_D & + \end{matrix}\right)$$

Force of
HIV infection
$$(F2)$$
 $(F2)$ $(F2)$ $(F2)$ $(F2)$ $(F2)$ $(F2)$ $(F2)$ $(F2)$ $(F2)$ $(F3)$ $(F2)$ $(F2)$ $(F3)$ $(F2)$ $(F2)$ $(F3)$ $(F2)$ $(F2)$ $(F2)$ $(F2)$ $(F2)$ $(F2)$ $(F2)$ $(F3)$ $(F2)$ $(F2)$ $(F2)$ $(F2)$ $(F2)$ $(F2)$ $(F2)$ $(F2)$ $(F2)$ $(F3)$ $(F3)$ $(F3)$ $(F2)$ $(F3)$ $(F2)$ $(F3)$ $(F2)$ $(F2)$ $(F2)$ $(F3)$ $(F3)$ $(F3)$ $(F2)$ $(F3)$ $(F2)$ $(F2)$ $(F3)$ $(F3$

Change in F4 infecteds on

$$\frac{dI_{F4}^{MT}}{dt} = \widetilde{\tau}_{F3}^{MT} I_{F3}^{MT} + \widetilde{\eta}_{F4} I_{F4}^{M} - \begin{pmatrix} Background & Drug-related & Exit treatment (F4) \\ death & death & death & rate \\ \mu & + & \mu_D & + & \xi \end{pmatrix}$$

$$= \widetilde{\tau}_{F3}^{MT} I_{F3}^{MT} + \widetilde{\eta}_{F4} I_{F4}^{M} - \begin{pmatrix} Background & Drug-related & Exit \\ death & death & rate \\ \mu & + & \mu_D & + & \xi \end{pmatrix}$$

$$= \widetilde{\tau}_{F3}^{MT} I_{F3}^{MT} + \widetilde{\eta}_{F4} I_{F4}^{M} - \begin{pmatrix} Background & Drug-related & Exit \\ death & death & rate \\ \mu & + & \mu_D & + & \xi \end{pmatrix}$$

$$= \widetilde{\tau}_{F3}^{MT} I_{F3}^{MT} + \widetilde{\eta}_{F4} I_{F4}^{M} - \begin{pmatrix} Background & Drug-related & Exit \\ rate & death & death & rate \\ \mu & + & \mu_D & + & \xi \end{pmatrix}$$

Change in liver
failure infecteds
$$\frac{dI_{LF}^{M}}{dt} = \tau_{F4LF}^{M} I_{F4}^{M} - \left(\begin{array}{c} Background & Liver failure & population \\ death & related death \\ \mu & + \mu_{LF} & + \xi_{L} & + \tau_{LFHCC} & \tau_{LFLT} \end{array} \right) I_{LF}^{M}$$

$$\frac{dI_{HCC}^{M}}{dt} = \tau_{F4HCC}^{M} I_{F4}^{M} + \tau_{LFHCC}^{M} I_{LF}^{M} + \tau_{LFHCC}^{M} I_{LF}^{M} - \begin{pmatrix} Background & HCC & Leaving \\ death & related death & with liver disease & LT \\ \mu & + \mu_{HCC} & + \xi_{L} & + \tau_{HCCLT} \end{pmatrix} I_{HCC}^{M}$$

$$\frac{dI_{LT}^{M}}{dt} = \tau_{LFLT}^{M} I_{LF}^{M} + \tau_{HCCLT}^{M} I_{HCC}^{M} - \begin{pmatrix} Background & HCC & Leaving \\ \mu & + \mu_{HCC} & + \xi_{L} & + \tau_{HCCLT} \end{pmatrix} I_{HCC}^{M}$$

$$\frac{dI_{LT}^{M}}{dt} = \tau_{LFLT}^{M} I_{LF}^{M} + \tau_{HCCLT}^{M} I_{HCC}^{M} - \begin{pmatrix} Background & Liver transplant & Leaving \\ \mu & + \mu_{LT} & + \xi_{L} & \end{pmatrix} I_{LT}^{M}$$

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HIV-infected individuals





Change in infecteds (350<CD4<500) during 1st treatment

$$\frac{dI_{350_500_{1st}}^{M}}{dt} = \underbrace{\eta_{350_500}^{U}I_{350_500}^{M}I_{350_500}^{M}I_{350_500}^{M}I_{350_500}^{M}I_{4}^{M} + \underbrace{\psi_{C}I_{350_500_{1st}}^{A}}_{HCV} + \underbrace{\psi_{C}I_{350_500}^{A}I_{200_350}^{M}I_{200_350}^{M}I_{200_350}^{M}I_{200_350}^{M}I_{4}^{M}}_{(350$$

Change in infecteds (200<CD4<350) during 1st treatment

$$\frac{during 1st treatment}{dt} = \underbrace{\begin{array}{l} Commenced 1st line \\ therapy (200 < CD4 < 350) \\ Undetectable Viral Load \\ dt \end{array}}_{Undetectable Viral Load } Spontaneous HCV \\ (acute, 200 < CD4 < 350) \\ (acute, 200 < CD4 < 350) \\ (CD4 < 200) \\ (CD4 < 350) \\ (CD4 <$$

Change in treatment failure infecteds (CD4>500)

$$\frac{dI_{500_{Fail}}^{M}}{dt} = \eta_{500}^{D}I_{500}^{M} + \psi_{C}I_{500_{Fail}}^{A} + \phi_{500}I_{500_{1st}}^{M} + \phi_{500}I_{500_{1st}}^{M} + \phi_{500}I_{500_{2nd}}^{M} - \left(\begin{matrix} Background during 2nd line therapy during 2nd line the$$

Change in treatment failure infecteds (350<CD4<500)

$$\frac{dI_{350-CD4}^{M}(350-CD4+500)}{dt} = \underbrace{\prod_{herapy}^{D}(350-CD4+500)}_{Detectable Viral Load} \\ = \underbrace{\prod_{herapy}^{D}(350-CD4+500)}_{Detectable Viral Viral Viral Viral Viral rebound} \\ = \underbrace{\prod_{herapy}^{D}(350-CD4+500)}_{Detectable Viral Load} \\ = \underbrace{\prod_{herapy}^{D}(350-CD4+500)}_{Detectable Viral Vi$$

$$\frac{dI_{200_350_{Fail}}^{M}}{dt} = \frac{\int_{200_350}^{D} I_{200_350}^{M}}{\int_{350_500}^{D} I_{350_500_{Fail}}^{M}} + \underbrace{\psi_{C}I_{200_350}^{A}}{\int_{40}^{D} I_{200_350}^{M}} + \underbrace{\psi_{C}I_{200_350_{Fail}}^{A}}{\int_{40}^{D} I_{200_350}^{M}} + \underbrace{\psi_{C}I_{200_350_{Fail}}^{A}}{\int_{40}^{D} I_{200_350}^{M}} + \underbrace{\psi_{C}I_{200_350_{Fail}}^{A}}{\int_{40}^{D} I_{200_350}^{M}} + \underbrace{\psi_{C}I_{200_350}^{A}}{\int_{40}^{D} I_{200_350}^{M}} + \underbrace{\psi_{C}I_{200_350}^{A}} + \underbrace{\psi_{C}I_{200_350}^{A}}{\int_{40}^{D} I_{200_350}^{M}} + \underbrace{\psi_{C}I_{200_350}^{A}}{\int_{40}^{D} I_{200_350}^{M}} + \underbrace{\psi_{C}I_{200_350}^{A}} + \underbrace{\psi_{C}I_{$$

$$+\omega_{200_350}^{D}+\sigma_{200_350}$$
 $\int I_{200_350_{Fail}}^{M}$

Change in treatment failure infecteds (CD4<200)

$$\frac{dI_{200_{Fail}}^{M}}{dt} = \underbrace{\eta_{200}^{D}I_{200}^{M}}_{P_{200}} + \underbrace{\psi_{C}I_{200_{Fail}}^{A}}_{P_{200}I_{200_{Ist}}} + \underbrace{\phi_{200}I_{200_{Ist}}^{M}}_{P_{200}I_{200_{Ist}}} + \underbrace{\phi_{200}I_{200_{Ist}}^{M}}_{Q_{200}I_{200_{Ist}}} + \underbrace{\phi_{200}I_{200_{Ist}}^{M}}_{Q_{200}I_{Ist}} + \underbrace{\phi_{200}I_{200_{Ist}}^{M}}_{Q_{200}I_{Ist}} + \underbrace{\phi_{200}I_{Ist}^{M}}_{Q_{200}I_{Ist}} + \underbrace{\phi_{200}I_{I$$

$$\frac{dI_{500_{2nd}}^{M}}{dt} = \sigma_{500}I_{500_{Fail}}^{M} + \psi_{C}I_{500_{2nd}}^{A} + \omega_{350_{500}}^{U}I_{350_{500}}^{M}I_{350_{2nd}}^{M}$$

$$= \sigma_{500}I_{500_{Fail}}^{M} + \psi_{C}I_{500_{2nd}}^{A} + \omega_{350_{500}}^{U}I_{350_{500}_{2nd}}^{M}$$

$$= \sigma_{500}I_{500_{Fail}}^{M} + \psi_{C}I_{500_{2nd}}^{A} + \omega_{350_{500}}^{U}I_{350_{500}_{2nd}}^{M}$$

$$= \sigma_{500}I_{500_{Fail}}^{M} + \omega_{D}^{U} + \xi + \omega_{350_{500}}^{U}I_{350_{2nd}}^{M}$$

$$= \sigma_{500}I_{500_{Fail}}^{M} + \omega_{D}^{U} + \xi + \omega_{350_{500}}^{U}I_{350_{2nd}}^{M}$$

$$\frac{dI_{350_500_{2nd}}^{M}}{dt} = \sigma_{350_500}I_{350_500}I_{350_500}I_{350_500}F_{ail}}^{M} + \widetilde{\psi}_{C}I_{350_500_{2nd}}^{A} + \widetilde{\psi}_{C}I_{350_50_{2nd}}^{A} + \widetilde{\psi}_{C}I$$

Change in infecteds (200<CD4<350) on 2nd line treatment

$$\frac{dI_{200_{350_{3nd}}}^{M}}{dt} = \overline{\sigma_{200_{350}}^{200_{350}}I_{200_{350}}^{M}}_{icutes}^{M} + \overline{\psi}_{C}I_{200_{350_{2nd}}}^{4} + \overline{\psi}_{C}I_{200_{350_{2nd}}}^{2} + \overline{\omega}_{200}^{U}I_{200_{2nd}}^{M}}_{icutes}^{M} - \left(\begin{array}{c}Background\\death\\ \mu + \mu_{D}\\ \end{array}\right) \\
\frac{dI_{200_{350}}^{M}}{dt} = \overline{\sigma_{200_{350}}I_{200_{350}}^{M}}_{icutes}^{M} + \overline{\psi}_{C}I_{200_{350_{2nd}}}^{4} + \overline{\omega}_{200_{350_{2nd}}}^{U} + \overline{\omega}_{200_{200}}^{U}I_{200_{2nd}}^{M}}_{icutes}^{M} + \overline{\omega}_{D}\\ \frac{HIV-related}{death}_{icutes}^{Force of} + \overline{\lambda}_{HCV} + \overline{\omega}_{200_{350}}^{S} + \overline{\omega}_{200_{350}}^{U}I_{200_{350}}^{M} + \overline{\omega}_{D}\\ \frac{HIV-related}{death}_{icutes}^{C} (200 < CD4 < 350) + HCV infection + \overline{\omega}_{200_{350}}^{C} + \overline{\omega}_{200_{350}}^{U}I_{200_{350}}^{M} + \overline{\omega}_{D}\\ \frac{HIV-related}{death}_{icutes}^{C} (200 < CD4 < 350) + \overline{\lambda}_{HCV} + \overline{\omega}_{200_{350}}^{S} + \overline{\omega}_{200_{350}}^{U}I_{200_{350}}^{M} + \overline{\omega}_{200_{350}}^{U}I_{200_{350}}^{M} + \overline{\omega}_{200_{350}}^{U}I_{200_{350}}^{M} + \overline{\omega}_{200_{350}}^{U}I_{200_{350}}^{M} + \overline{\omega}_{200_{350}}^{U}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M} + \overline{\omega}_{200_{360}}^{U}I_{200_{360}}^{M} + \overline{\omega}_{200_{360}}^{U}I_{200_{360}}^{M} + \overline{\omega}_{200_{360}}^{U}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{200_{360}}^{M}I_{2$$

The equations above describe the change in the number of people in each health state for HIV and HCV monoinfection. The complete mathematical model also includes equations for each possible HIV and HCV coinfection combination, with the terms of the ordinary differential equations amalgamating the appropriate HIV and HCV terms.

Assumptions for modelling secondary transmissions

The average age at which IDUs in Australia acquire HCV infection is their early to mid 20s [53, 67, 68] and it is likely that IDUs who acquire HIV infection would do so a few years thereafter on average. Approximately p_{MSM} =65% of IDUs with HIV are men who have sex with men, p_{F} =7% are females and the remaining p_{M} =28% are heterosexual males [54]. For IDUs with HCV infection, approximately p_{MSM} =5% of IDUs with HIV are men who have sex with men, p_{F} =32% are females and the remaining p_{M} =63% are heterosexual males [54]. It is assumed that the average number of long-term sexual partners each IDU has after acquiring infection is c_{reg} =3-5. In addition, it is assumed that each heterosexual IDU would have an

average of c_{cas} =5-10 casual short-term sexual partners (each with one penetrative act) after acquiring infection and male homosexual IDUs would have an average of c_{cas} =20-30 casual partners after acquiring infection. Most IDUs who share syringes tend to do so with sexual partners or close friends [69, 70]. Therefore, some of the potential partners are not susceptible to transmission because they are already infected (and probably the primary source for the case in question); this complexity is not considered here. It is assumed that HIV transmission rates are β_{HL} =0.01 for heterosexual transmission for long-term partnerships, β_{ML} =0.1 for male homosexuals for long-term partnerships, and per-act probabilities of HIV transmission during casual partnerships are β_{FM} =0.0005 for female-to-male transmission, β_{MF} =0.001 for male-tofemale transmission, and β_{MM} =0.01 for male-to-male transmission during unprotected sex [71-77]. It is assumed that condom usage is *q*=80% [78], with efficacy of 95% [79-83]. HCV transmission per sexual contact is assumed to be β_{HCV-a} =0.1% per act and β_{HCV-p} =2% per long-term partnership [84-86].

The average fertility rate in Australia is f=1.93 babies per woman over her lifetime [87] and the median age of all mothers of births is ~31 years [88]. Based on the average age at infection and the relatively similar infection ages for HIV and HCV, it is assumed that 75% of a woman's births occur after she acquires infection [88]. The probability of mother-tochild transmission is $\beta_{\text{HIV-MTCT}}=2\%$ for HIV (with the use of antiretrovirals and Caesarean section) [89-92] and $\beta_{\text{HCV-MTCT}}=5\%$ for HCV [93-95].

Therefore, the average number of secondary infections through sexual transmission or mother-to-child transmission per HIV infection is

$$s_{HIV} = p_{MSM} \left(c_{reg} \beta_{ML} + c_{cas} \beta_{MM} \right) + p_M \left(c_{reg} \beta_{HL} + c_{cas} \beta_{MF} \right)$$
$$+ p_F \left(c_{reg} \beta_{HL} + c_{cas} \beta_{FM} + 0.75 f \beta_{HIV-MTCT} \right)$$

and the average number of secondary HCV transmissions expected per HCV infection is

$$s_{HCV} = p_{MSM} \left(c_{reg} \beta_{HCV-p} + c_{cas} \beta_{HCV-a} \right) + p_M \left(c_{reg} \beta_{HCV-p} + c_{cas} \beta_{HCV-a} \right)$$
$$+ p_F \left(c_{reg} \beta_{HCV-p} + c_{cas} \beta_{HCV-a} + 0.75 f \beta_{HCV-MTCT} \right)$$

Substituting parameter estimates leads to 0.44 and 0.11 secondary HIV and HCV cases, respectively, for each primary infection.